

性质 24.

Pf: (1) 记 $E = \{a_n\}$ 的所有极限点

$$\limsup_{n \rightarrow \infty} a_n = \sup E, \quad \liminf_{n \rightarrow \infty} a_n = \inf E$$

$$\Rightarrow \liminf_{n \rightarrow \infty} a_n \leq \limsup_{n \rightarrow \infty} a_n$$

$$(2) \lim_{n \rightarrow \infty} a_n = a \Leftrightarrow E = \{a\} \Leftrightarrow \limsup_{n \rightarrow \infty} a_n = \liminf_{n \rightarrow \infty} a_n = a.$$

$$(3). \text{ Pf: } a^* = \limsup_{n \rightarrow \infty} a_n, \quad b^* = \limsup_{k \rightarrow \infty} b_k \Rightarrow \text{w.t.p. } a^* \leq b^*$$

$$(1) b^* = +\infty \quad \checkmark$$

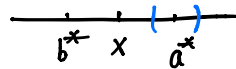
$$a^* = +\infty, \quad \lim_{k \rightarrow \infty} a_{n_k} = +\infty$$

$$k \text{ 充分大, } a_{n_k} \leq b_{n_k} \Rightarrow b^* = +\infty$$

$$a^* = -\infty \quad \checkmark, \quad b^* = -\infty$$

(2). a^*, b^* 有限数

反证: 若 $a^* > b^*$



$$n > N_1, \quad b_n \leq x$$

$$n > \max\{N, N_1\}, \quad \underline{a_n \leq b_n \leq x < a^*}$$

$\{a_n\}$ 中除有限项外有上界 x

\therefore 上极限 $a^* \leq x$, 而 $x < a^*$ 矛盾.