

例 6:

证: 对点列 $\{x_n\}$: $x_n \in D$, $x_n \neq x_0$ 且 $\lim_{n \rightarrow \infty} x_n = x_0$.

由数列上下极限性质 3.2 知

$$\liminf_{n \rightarrow \infty} f(x_n) + \liminf_{n \rightarrow \infty} g(x_n) \leq \limsup_{n \rightarrow \infty} (f(x_n) + g(x_n)) \leq \limsup_{n \rightarrow \infty} f(x_n) + \limsup_{n \rightarrow \infty} g(x_n)$$

$$\text{又 } \min \{ \liminf_{n \rightarrow \infty} f(x_n) \} \leq \max \{ \liminf_{n \rightarrow \infty} f(x_n) \}$$

由定理 4.2 知

$$\liminf_{x \rightarrow x_0} f(x) + \limsup_{x \rightarrow x_0} g(x) \leq \limsup_{x \rightarrow x_0} (f(x) + g(x)) \leq \limsup_{x \rightarrow x_0} f(x) + \lim_{x \rightarrow x_0} g(x).$$

$$\text{同理可证: } \liminf_{x \rightarrow x_0} f(x) + \liminf_{x \rightarrow x_0} g(x) \leq \liminf_{x \rightarrow x_0} (f(x) + g(x)) \leq \liminf_{x \rightarrow x_0} f(x) + \limsup_{x \rightarrow x_0} g(x)$$